Report of Research
Bass Reflex Speaker System(1)

by Shigeru Suzuki
at 103rd Sandokai Meeting
of Tezukiri Amp no Kai

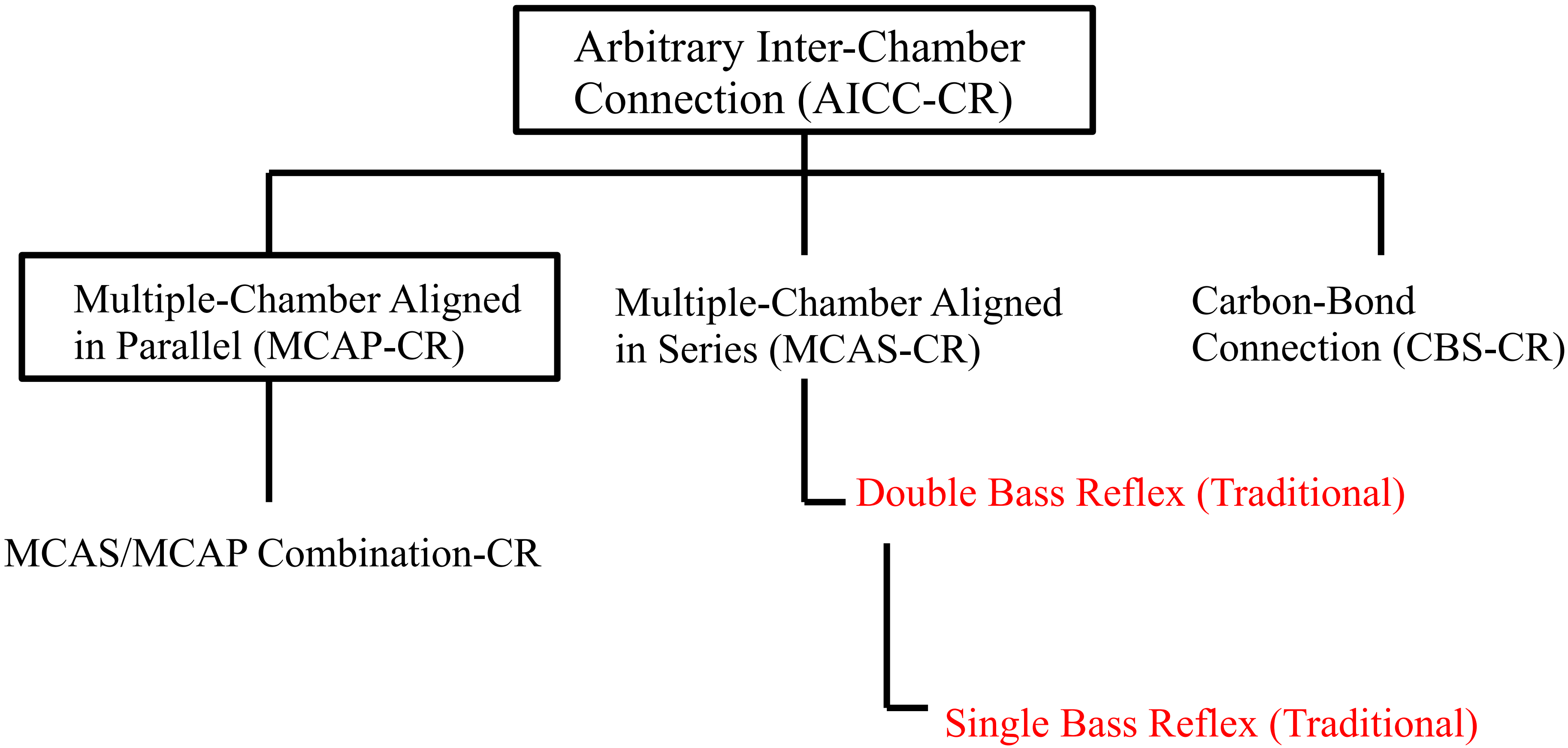
What is Bass-Reflex System? (1)

- Most common speaker enclosure in the world.
- General Definition#1: Enclosure consists of Helmholtz's Resonator or Cavity Resonator
- General Definition#2: Phase of displacement of air in duct is opposite to speaker membrane. Movement of air in the duct reinforces low frequency.
- Suzuki's definition: Speaker enclosure that consists of masses and springs. Chambers act as spring and air in ducts act as masses.
- Cavity resonator is different from pipe resonator. Cavity resonator assumes that air in the duct act as single mass, while pipe resonator assumes air in the pipe does not act as single mass. Air in pipe resonator has dense layers and less dense layers. Hence very long duct in bass reflex enclosure acts as pipe resonator.

What is Bass-Reflex System? (2)

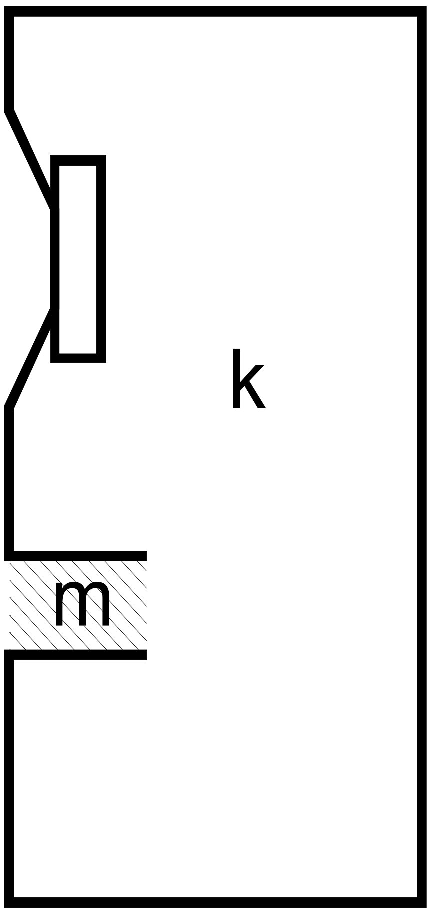
- Chamber & duct acts as frequency filter (you will see more details later).
- Characteristic frequencies can be calculated from free vibration equations of motion derived from state equation of gas.
  - Adiabatic Condition (Common theory)
  - Equithermal Condition (Suzuki's opinion)
  - Existing formulas are closer to equithermal condition (i.e. Nagaoka's books)
- It is general system, but is not understood well.
- Designing theory of even single bass reflex system is still incomplete (Suzuki's opinion)
- 2 DOF (Double Bass Reflex) systems are well developed; however, it is believed that calculation is too difficult.
- Number of degrees of freedom can be as many as we want, but multiple DOF system has not been well studied yet.

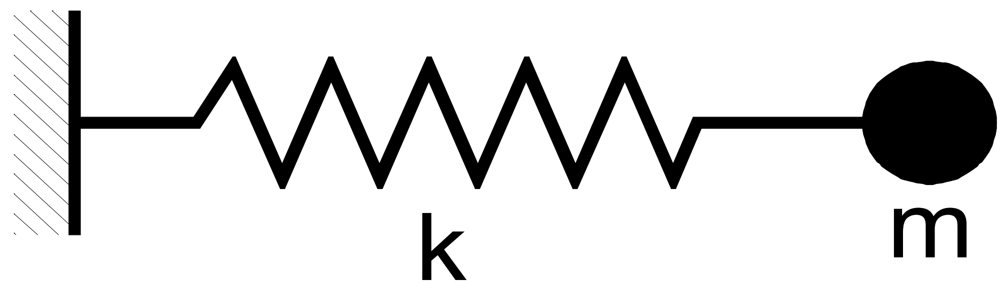
Classification of MDOF System



These systems are named by Suzuki except red texts.
There are a number of bass reflex systems; just not studied enough

1Degree of Freedom System

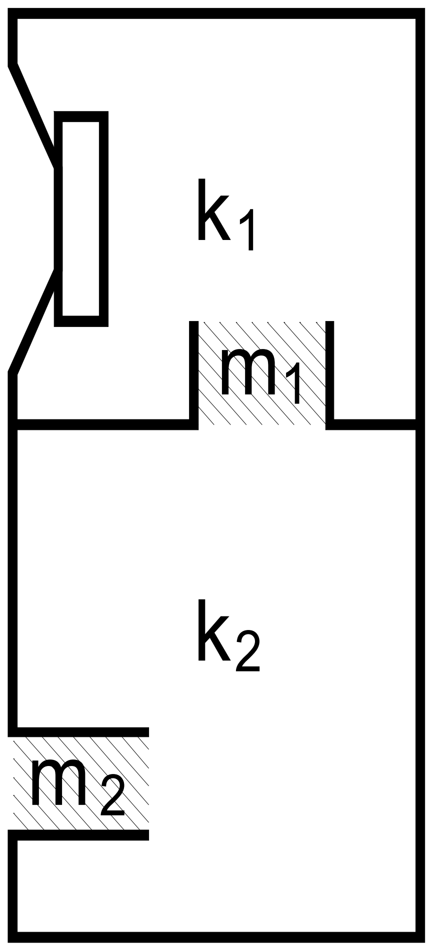


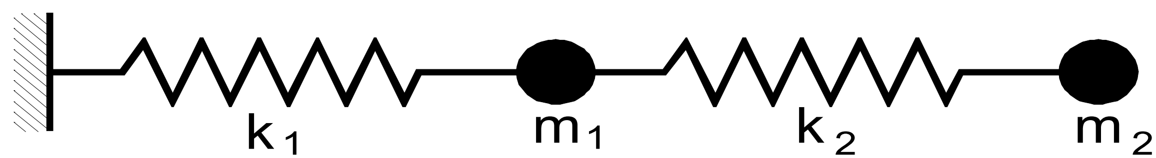


$$f_D = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Add one more degree of freedom (mass of speaker membrane) .

2Degree of Freedom System

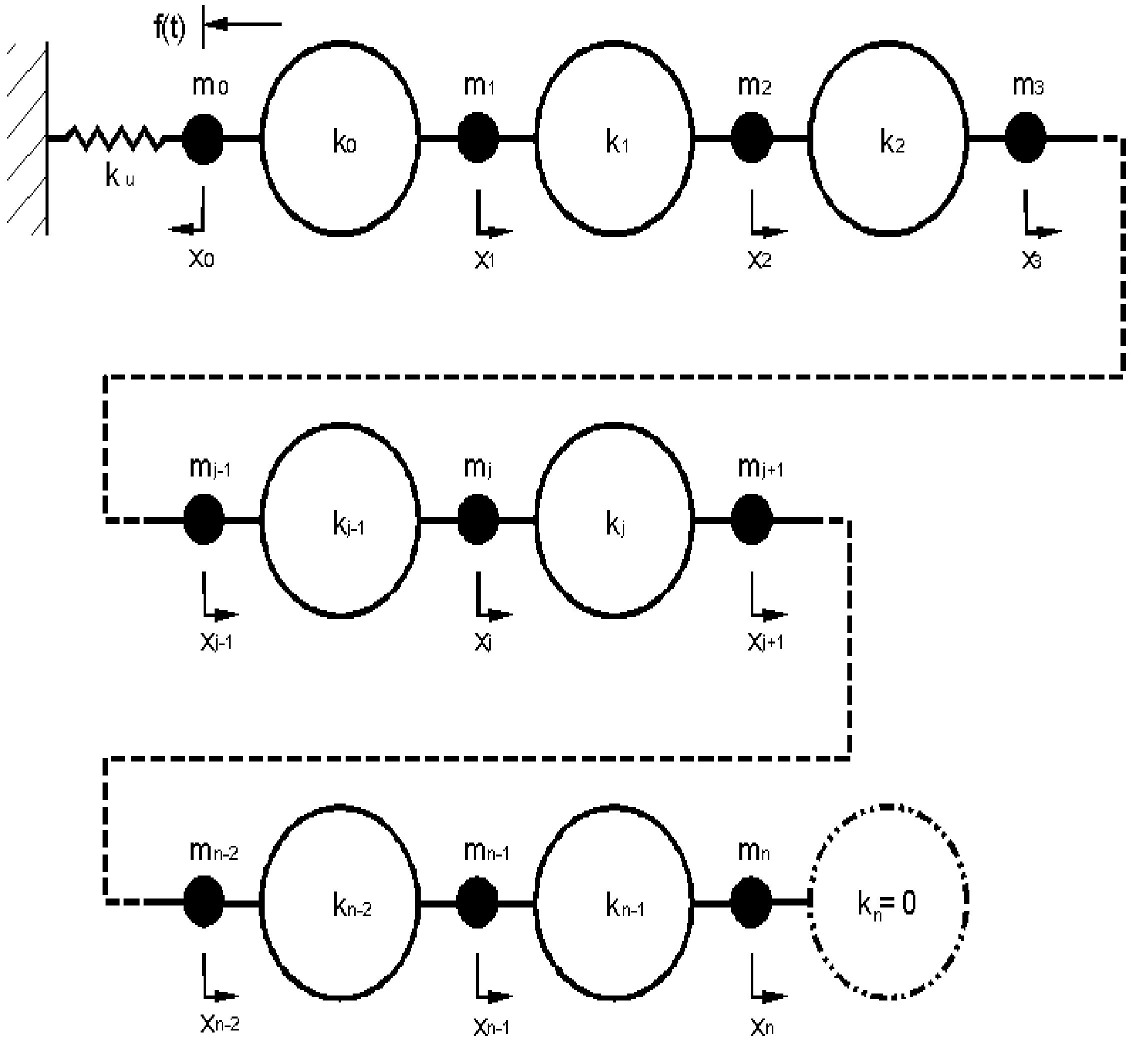




$$f_D = \frac{1}{2\pi}\sqrt{\frac{k_{11}m_2 + k_{22}m_1 \pm \sqrt{\left(k_{11}m_2 + k_{22}m_1\right)^2 - 4m_1m_2\left(k_{11}k_{22} - k_{12}k_{21}\right)}}{2m_1m_2}}$$

Add one more degree of freedom (mass of speaker membrane) .

3 or More Degree of Freedom System (MCAS-CR)

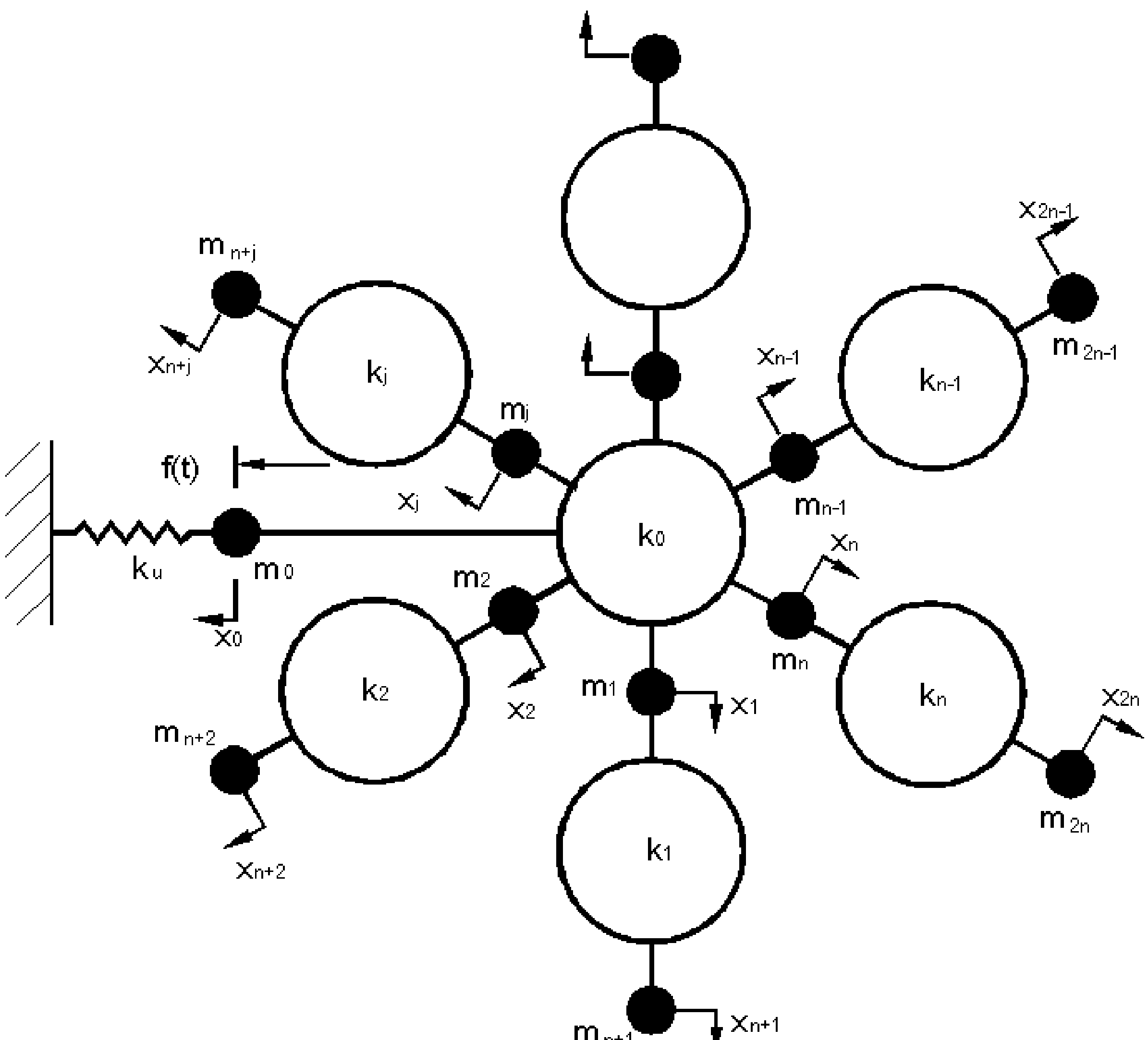


Multiple-Chamber Aligned in Series

Equation of Motion

$$
\begin{cases}
m_0\ddot{x}_0 + (k_u + k_0)x_0 + k_0 r_1 x_1 = f(t) \\
m_1\ddot{x}_1 + k_0 r_1 x_0 + (k_0 + k_1) r_1^2 x_1 - k_1 r_1 r_2 x_2 = 0 \\
m_2\ddot{x}_2 - k_1 r_1 r_2 x_1 + (k_1 + k_2) r_2^2 x_2 - k_2 r_2 r_3 x_3 = 0 \\
\ldots\ldots \\
m_j\ddot{x}_j - k_{j-1} r_{j-1} x_{j-1} + (k_{j-1} + k_j) r_j^2 x_j - k_j r_j r_{j+1} x_{j+1} = 0 \\
\ldots\ldots \\
m_{n-1}\ddot{x}_{n-1} - k_{n-2} r_{n-2} x_{n-1} + (k_{n-2} + k_{n-1}) r_{n-1}^2 x_{n-1} - k_{n-1} r_{n-1} r_n x_n = 0 \\
m_n\ddot{x}_n - k_{n-1} r_{n-1} r_n x_{n-1} + (k_{n-1} + k_n) r_n^2 x_n = 0
\end{cases}
$$

3 or More Degree of Freedom System (MCAP-CR)

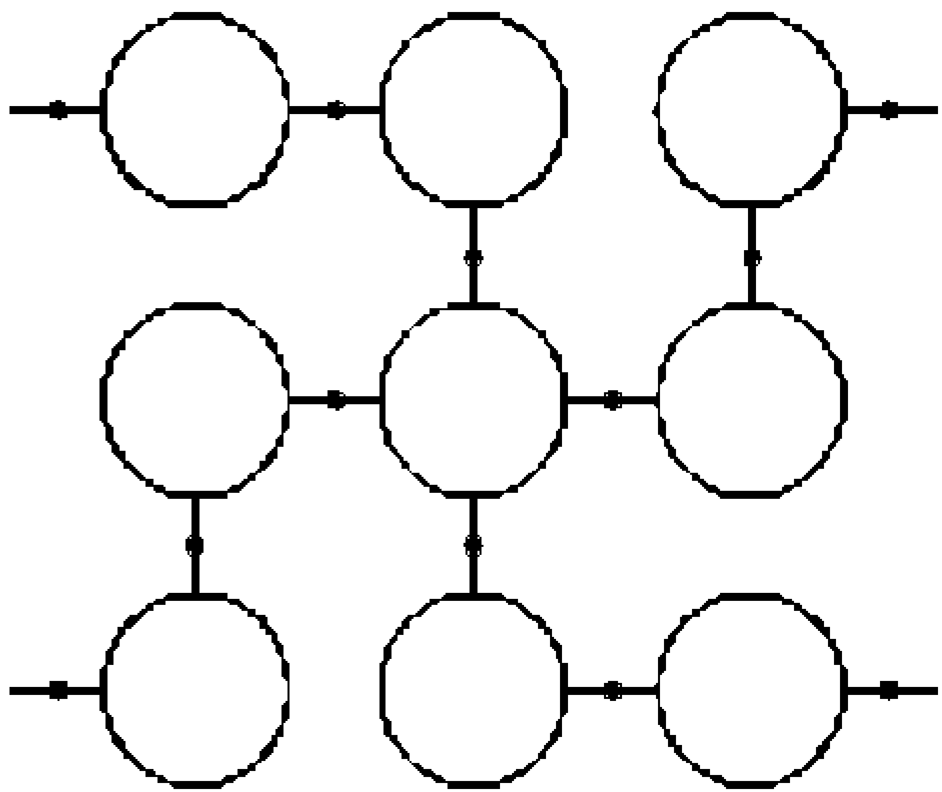


Multiple-Chamber Aligned in Parallel

Equation of Motion

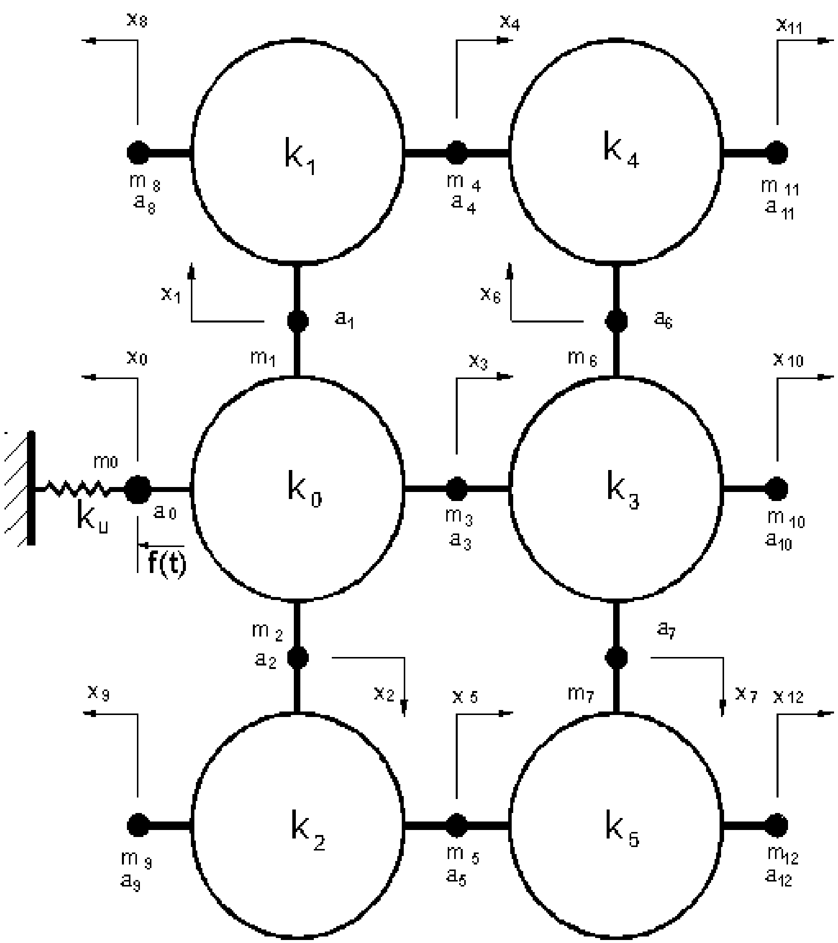
$$
\begin{cases}
m_0 \dfrac{d^2 x_0}{dt^2} + \left(k_u + k_0 r_0^2\right) x_0 + k_0 \sum_{i=1}^{N} r_0 r_i x_i = f(t) \\
m_j \dfrac{d^2 x_j}{dt^2} + k_0 r_j \sum_{i=0}^{N} r_i x_i + k_j r_j \left(r_j x_j - r_{j+N} x_{j+N}\right) = 0 \\
m_{j+N} \dfrac{d^2 x_{j+N}}{dt^2} - k_j r_{j+N} \left(r_j x_j - r_{j+N} x_{j+N}\right) = 0
\end{cases}
$$

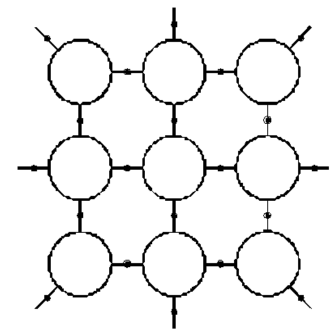
3 or More Degree of Freedom System (MCAS&MCAS Combination-CR)



MCAS/MCAP Combination

3 or More Degree of Freedom System (CBS-CR)

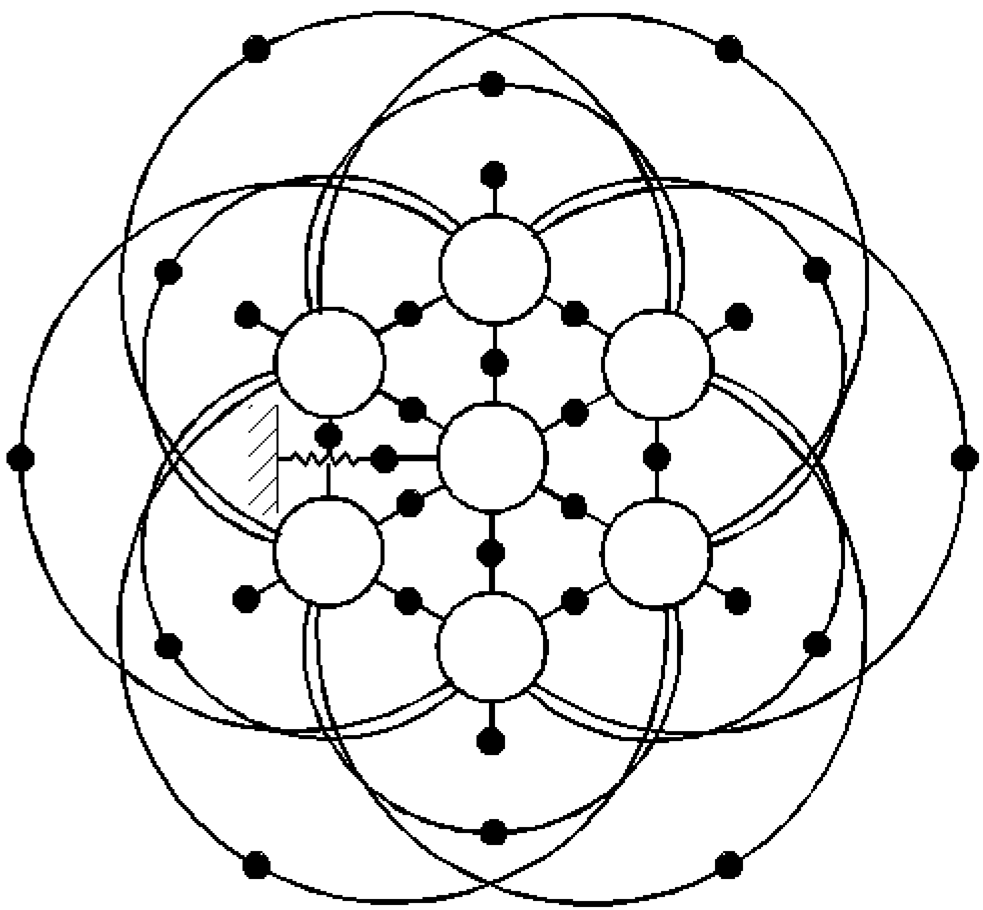




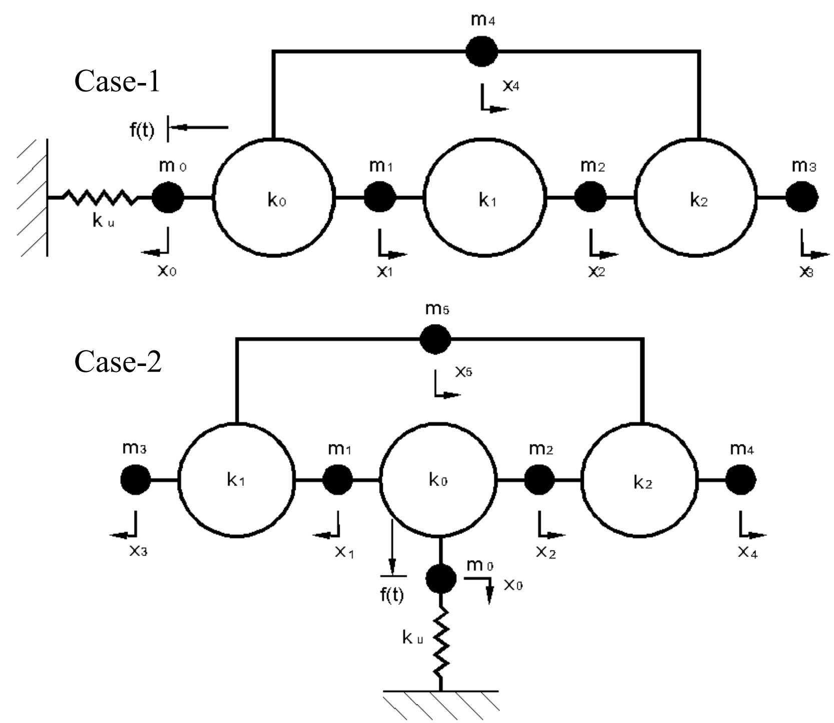
$$
\begin{cases}
m_0\ddot{x}_0 + \left(k_u + k_0 r_0^2\right)x_0 + k_0 r_0\left(r_1 x_1 + r_2 x_2 + r_3 x_3\right) = f(t) \\
m_1\ddot{x}_1 + k_0 r_1\left(r_0 x_0 + r_1 x_1 + r_2 x_2 + r_3 x_3\right) + k_1 r_1\left(r_1 x_1 - r_4 x_4 - r_8 x_8\right) = 0 \\
m_2\ddot{x}_2 + k_0 r_2\left(r_0 x_0 + r_1 x_1 + r_2 x_2 + r_3 x_3\right) + k_2 r_2\left(r_2 x_2 - r_5 x_5 - r_9 x_9\right) = 0 \\
m_3\ddot{x}_3 + k_0 r_3\left(r_0 x_0 + r_1 x_1 + r_2 x_2 + r_3 x_3\right) + k_3 r_3\left(r_3 x_3 - r_6 x_6 - r_7 x_7 - r_{10} x_{10}\right) = 0 \\
m_4\ddot{x}_4 + k_1 r_4\left(- r_1 x_1 + r_4 x_4 + r_8 x_8\right) + k_4 r_4\left(r_4 x_4 + r_6 x_6 - r_{11} x_{11}\right) = 0 \\
m_5\ddot{x}_5 + k_2 r_5\left(- r_2 x_2 + r_5 x_5 + r_9 x_9\right) + k_5 r_5\left(r_5 x_5 + r_7 x_7 - r_{12} x_{12}\right) = 0 \\
m_6\ddot{x}_6 + k_3 r_6\left(- r_3 x_3 + r_7 x_7 + r_{10} x_{10}\right) + k_4 r_6\left(r_4 x_4 + r_6 x_6 - r_{11} x_{11}\right) = 0 \\
m_7\ddot{x}_7 + k_3 r_7\left(- r_3 x_3 + r_6 x_6 + r_{10} x_{10}\right) + k_5 r_7\left(r_5 x_5 + r_7 x_7 - r_{12} x_{12}\right) = 0 \\
m_8\ddot{x}_8 + k_1 r_8\left(- r_1 x_1 + r_4 x_4 + r_8 x_8\right) = 0 \\
m_9\ddot{x}_9 + k_2 r_9\left(- r_2 x_2 + r_5 x_5 + r_9 x_9\right) = 0 \\
m_{10}\ddot{x}_{10} + k_3 r_{10}\left(- r_3 x_3 + r_6 x_6 + r_7 x_7 + r_{10} x_{10}\right) = 0 \\
m_{11}\ddot{x}_{11} + k_4 r_{11}\left(- r_4 x_4 - r_6 x_6 + r_{11} x_{11}\right) = 0 \\
m_{12}\ddot{x}_{12} + k_5 r_{12}\left(- r_5 x_5 - r_7 x_7 + r_{12} x_{12}\right) = 0
\end{cases}
$$

Carbon Bond Structured

3 or More Degree of Freedom System (AICC-CR)



Arbitrary Inter-Chamber Connection



$$m_1 \frac{d^2 x_1}{dt^2} + k_0 r_1^2 x_1 + k_0 r_1 r_2 x_2 + k_0 r_1 r_0 x_0 - k_1 r_1 r_3 x_3 - k_1 r_1 r_5 x_5 = 0$$

$$m_2 \frac{d^2 x_2}{dt^2} + k_0 r_2^2 x_2 + k_0 r_2 r_1 x_1 + k_0 r_2 r_0 x_0 - k_2 r_2 r_4 x_5 + k_2 r_2 r_6 x_5 = 0$$

$$m_3 \frac{d^2 x_3}{dt^2} + k_1 r_3^2 x_1 + k_1 r_5 r_3 x_5 - k_1 r_1 r_3 x_1 = 0$$

$$m_4 \frac{d^2 x_4}{dt^2} + k_2 r_4^2 x_4 - k_2 r_5 r_4 x_5 - k_2 r_2 r_4 x_4 = 0$$

$$m_5 \frac{d^2 x_5}{dt_2} + (k_1 + k_2) r_5^2 x_5 + k_1 (r_5 r_3 x_3 - r_5 r_1 x_1) + k_2 (r_5 r_2 x_2 - r_5 r_4 x_4) = 0$$

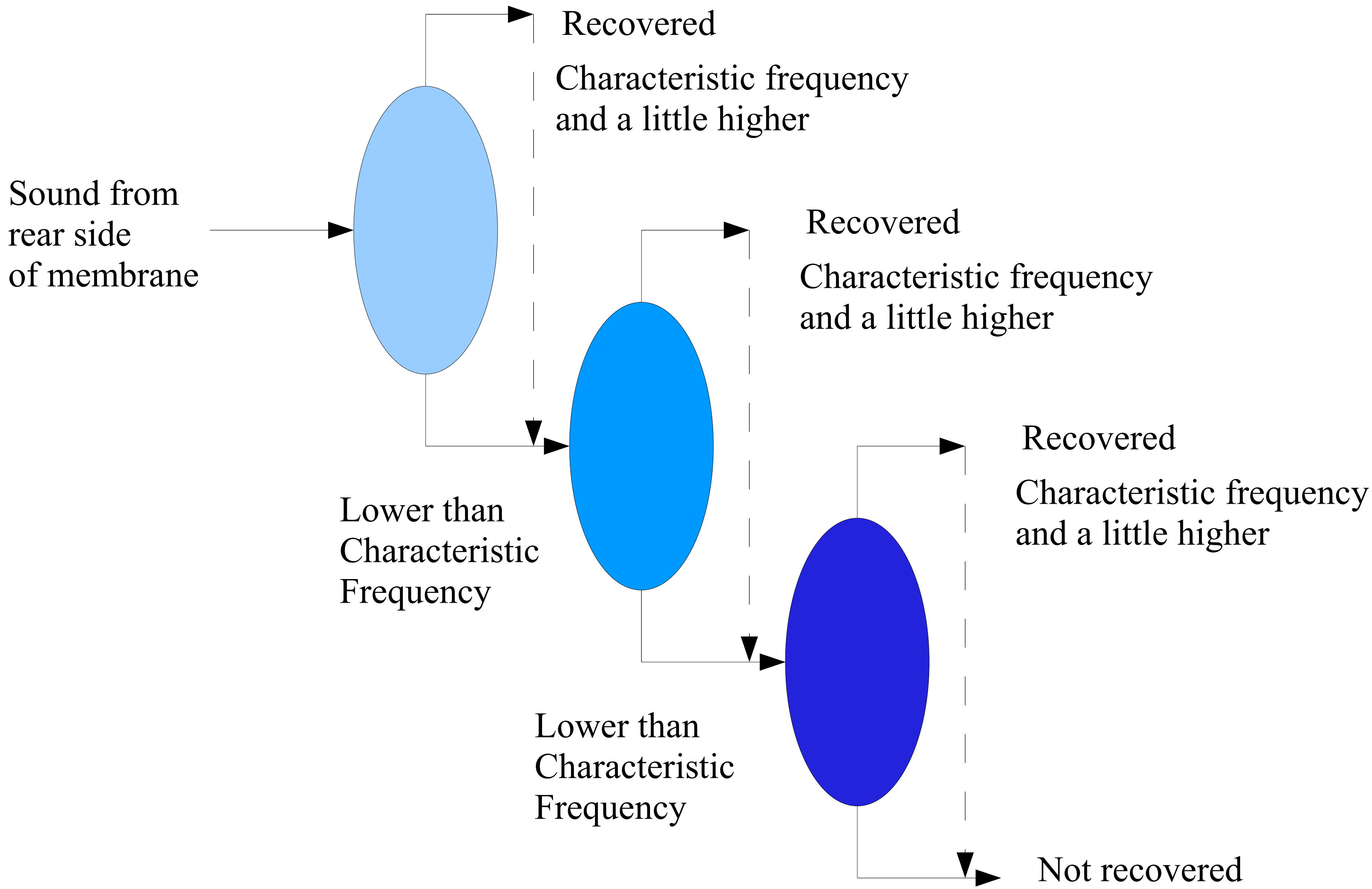
$$m_5 \frac{d^2 x_5}{dt_2} - k_1 r_5 r_1 x_1 + k_2 r_5 r_2 x_2 + k_1 r_5 r_3 x_3 - k_2 r_5 r_4 x_4 + (k_1 + k_2) r_5^2 x_5 = 0$$

3 or More Degree of Freedom System Summary

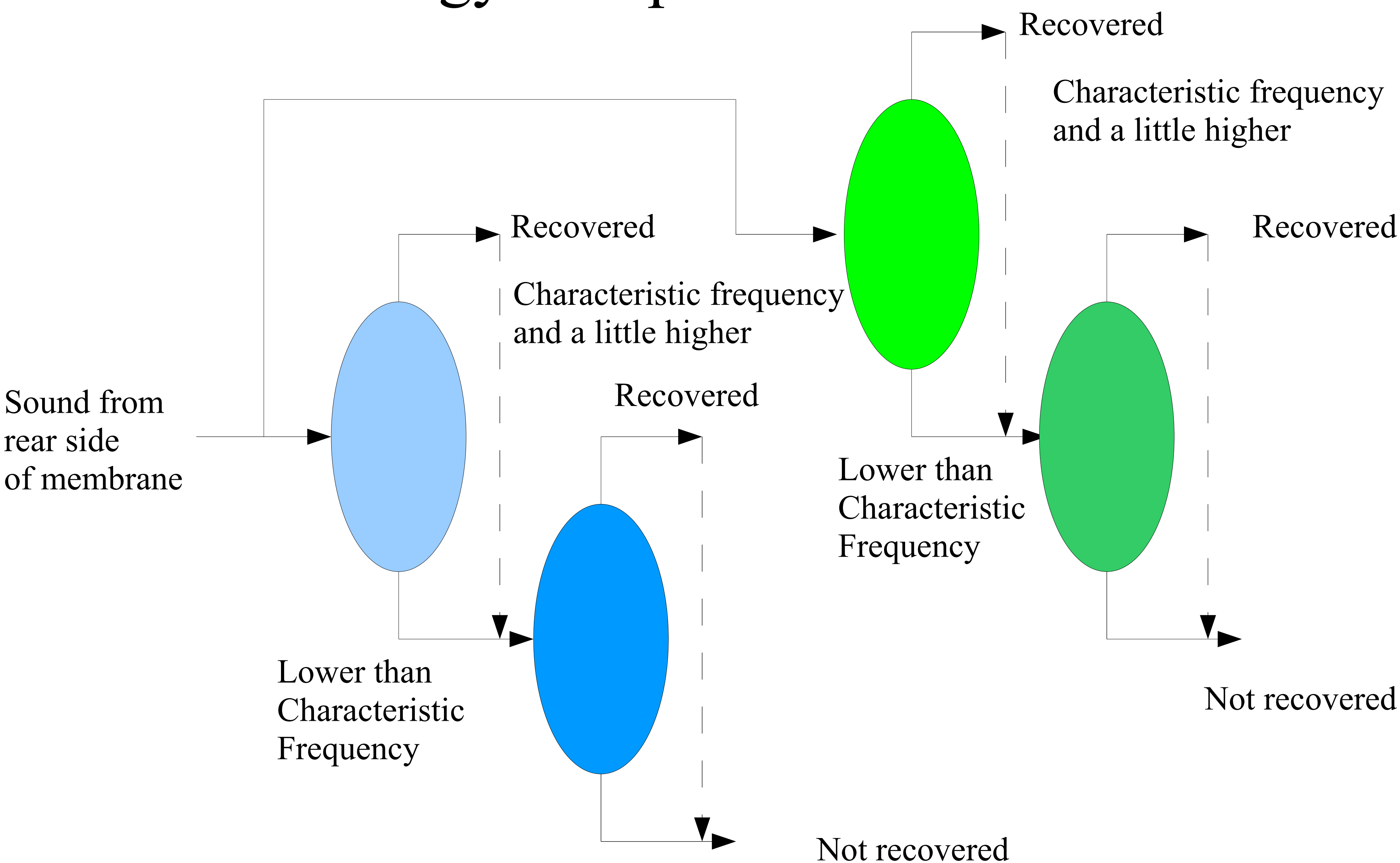
Case-1

- MCAS-CRs, MCAP-CRs and their combination systems have been modeled and equations of motion have been proposed by Suzuki.
- Characteristic frequencies of MCAS-CRs are easily calculated. Disadvantages of MCAS-CR are:
  - Numbers of degrees of freedom is less than MCAP-CRs where number of chambers is equivalent.
  - Sound from exposed duct may be too much filtered by number of chambers and ducts..
- Numbers of degrees of freedom of MCAP-CR is up to twice as many as number of chambers. Thus we may flexibly design MCAP-CRs.
- AICC-CRs and CBS-CRs could be calculated, but equations are complicated and also equations do not look smart.

Idea of MCAS-CR analogy to separation columns



Idea of MCAP-CR analogy to separation columns

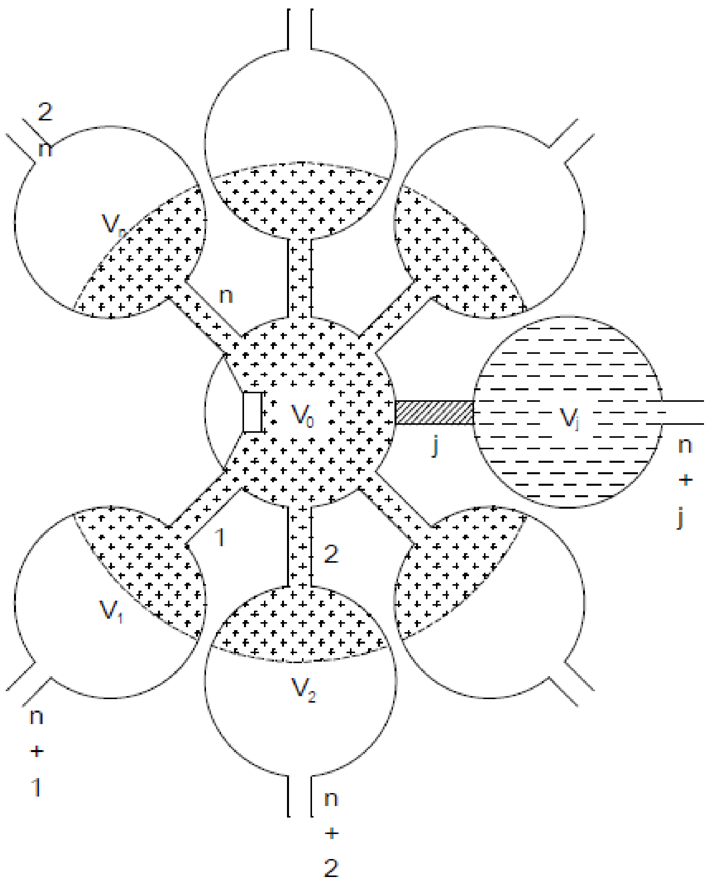


Calculation of Characteristic Frequencies of MDOF-CRs

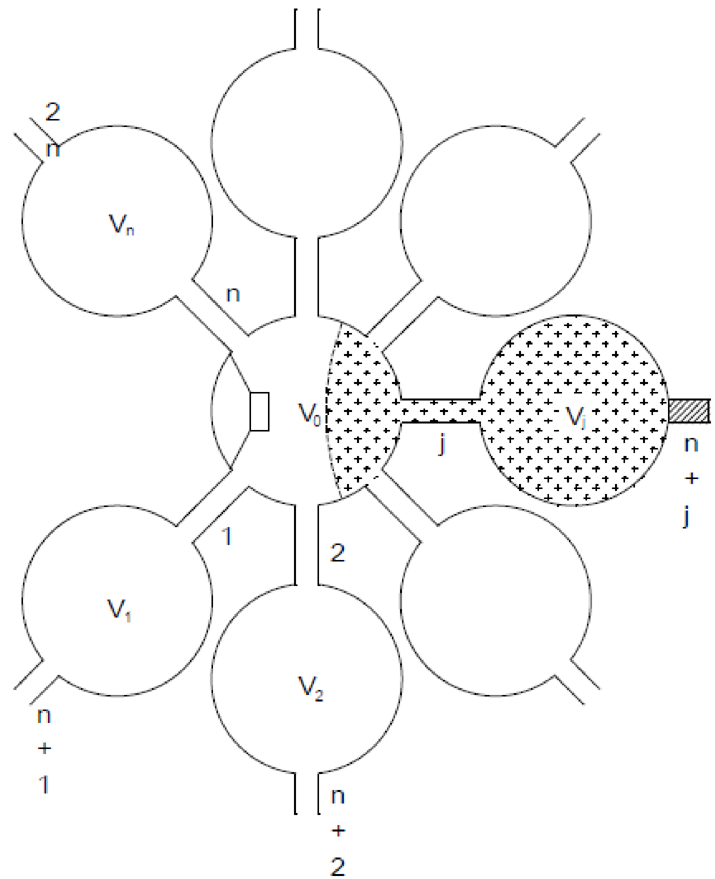
- Derive equations of motion for free vibration problem, then solve characteristic equations and calculate eigenvalues.
- Characteristic Equation:
  - $|K-\lambda M| = 0$ or $|M^{-1}K-\lambda I|=0$
- Calculation will become more difficult if number of DOF increases
  - $M^{-1}K$ is not symmetric matrix in most cases.
- Error of eigenvalue calculation is generally large and may not be small enough.
- Simplest method (not exact but practical):
  - Calculate $f(\lambda)=|K-\lambda M|$
  - $f_j=1/2\pi*\lambda_j$

Simplified Estimation Method of Characteristic Frequencies of MCAP-CR (1)

Internal ducts



Open-air ducts



- Add some of capacities of sub-chambers to capacity of main chamber
- Sub-chamber next to main chamber acts as reducing capacity of main chamber for reference duct.

- Add some of capacity of main chamber to capacity of reference sub-chamber.

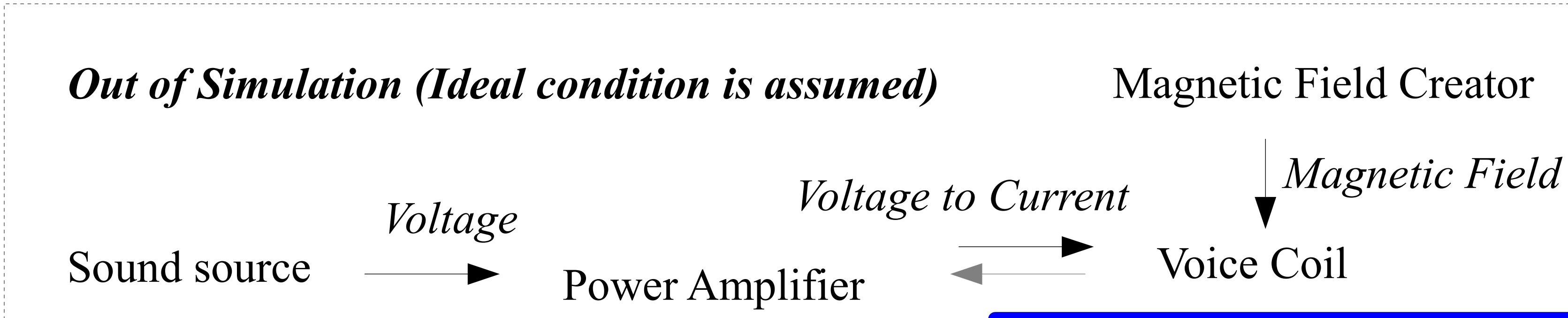
Simplified Estimation Method of Characteristic Frequencies of MCAP-CR (2)

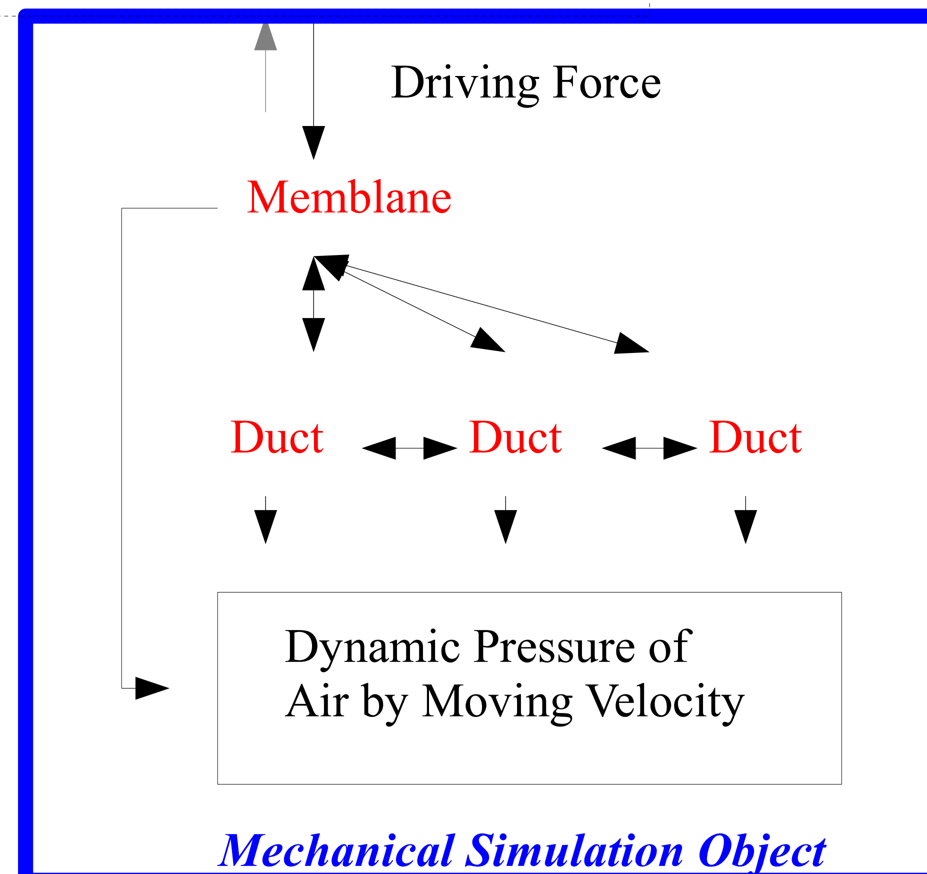
Table 2 Calculation Formulae of Characteristic Equations Based on Simplified Assumption

	Equivalent Volume[m³]	Equivalent Stiffness [N/m]	Mass[kg]	Characteristic Frequencies[Hz]
Inter -chamber duct j=1, ...,n	$\hat{V}_0 = V_0 + \sum_{i=0}^{n} \alpha_i V_i - V_j$ $\hat{V}_j = \dfrac{1}{\dfrac{1}{\hat{V}_0} + \dfrac{1}{V_j}}$	$\hat{k}_j^* = \dfrac{r_j^2 a_0^2 \gamma \cdot P}{\hat{V}_j}$	$m_j = \rho \cdot r_j a_0 l_j$	$\hat{f}_j = \dfrac{1}{2\pi}\sqrt{\dfrac{\hat{k}_j^*}{m_j}}$
Open -air duct j=1, ...,n	$\hat{V}_j = V_j + \beta_j V_0$	$\hat{k}_{j+n}^* = \dfrac{r_{j+n}^2 a_0^2 \gamma \cdot P}{\hat{V}_j}$	$m_{j+n} = \rho \cdot r_{j+n} a_0 l_{j+n}$	$\hat{f}_{j+n} = \dfrac{1}{2\pi}\sqrt{\dfrac{\hat{k}_{j+n}^*}{m_{j+n}}}$

This method does not consider effect of other ducts than reference duct.
Refer to document MCAP006E for more details.

Simulation of MDOF-CR System(0)





Assumptions:

Power amplifier amplifies voltage; when current is short, voltage drops: thus power amplifier loses linearity.

Ideal (perfectly linear) amplifier is assumed for simplicity.

Driving force is assumed to be proportional to driving voltage for simplicity (It is not realistic, because impedance is not uniform over frequency region.)

Simulation of MDOF-CR System(1)

- Difference formula of equation of motion of MDOF-CRs can be solved by recurrence formula. Initial conditions must be set.
- All MDOF-CRs (MCAS, MCAP, AICC, and CBS) can be simulated as above.
- Initial value problem as above may be solved using spreadsheet software like Open Office Calc. Note that calculation of spreadsheet software is very slow so that compiled program is more ideal.
- Sound pressure may be derived from calculated velocity of mass. Velocity of mass is calculated from displacement and discretized time. Sound pressure = $1/2*\rho*v^2$ [Pa]. This value may be converted to dB.

Simulation of MDOF-CR System(2)

Equation of Motion of Forced Vibration in Matrix Form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(\mathbf{t})$$

Damping term is ignored for simplicity.

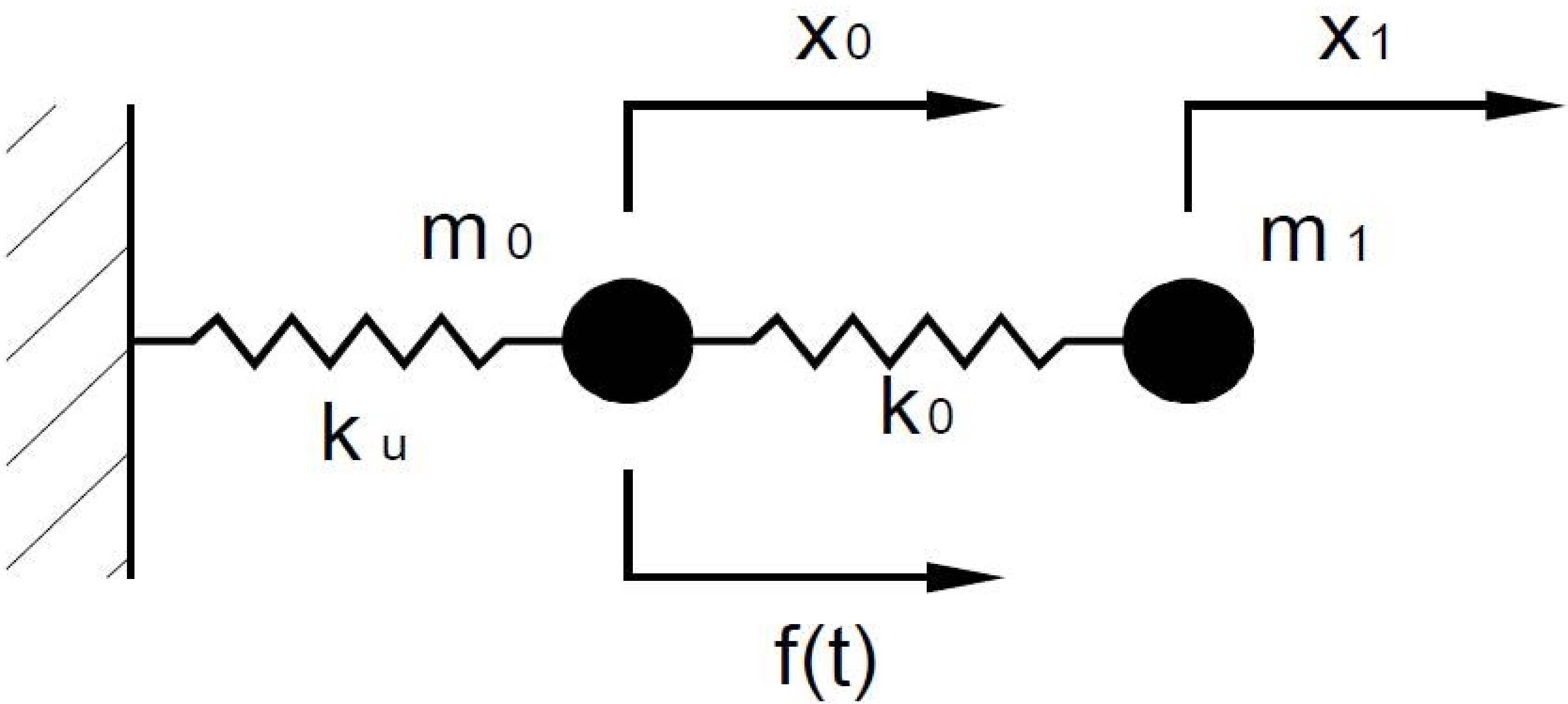
Discretized Equation of Motion in Central Difference Form

$$\mathbf{M}\frac{\mathbf{x}^{j+1} - 2\mathbf{x}^{j} + \mathbf{x}^{j-1}}{\delta^2} + \mathbf{C}\left(\frac{\mathbf{x}^{j+1} - \mathbf{x}^{j-1}}{2\delta}\right) + \mathbf{K}\mathbf{x}^{j} = \mathbf{f}(\omega \cdot \delta \cdot j)$$

Damping term is ignored for simplicity.

Simulation of MDOF-CR System(3)

Vibration Model of Single Bass Reflex Speaker System



Simulation of MDOF-CR System(4)

Reccurence form of discretized equation of motion

$$\begin{bmatrix} x_0^{j+1} \\ x_1^{j+1} \end{bmatrix} = \begin{bmatrix} 2 - \dfrac{\delta^2 (k_u + k_0)}{m_0} & \dfrac{\delta^2 r_1 k_0}{m_0} \\ \dfrac{\delta^2 k_0 r_1}{m_1} & 2 - \dfrac{\delta^2 r_1^2 k_0}{m_1} \end{bmatrix} \begin{bmatrix} x_0^{j} \\ x_1^{j} \end{bmatrix} - \begin{bmatrix} x_0^{j-1} \\ x_1^{j-1} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta^2}{m_0} f^j \\ 0 \end{bmatrix}$$

1 Step Ahead

Current Displacement of mass

1 Step before

Driving Force

Simulation of MDOF-CR System(5)

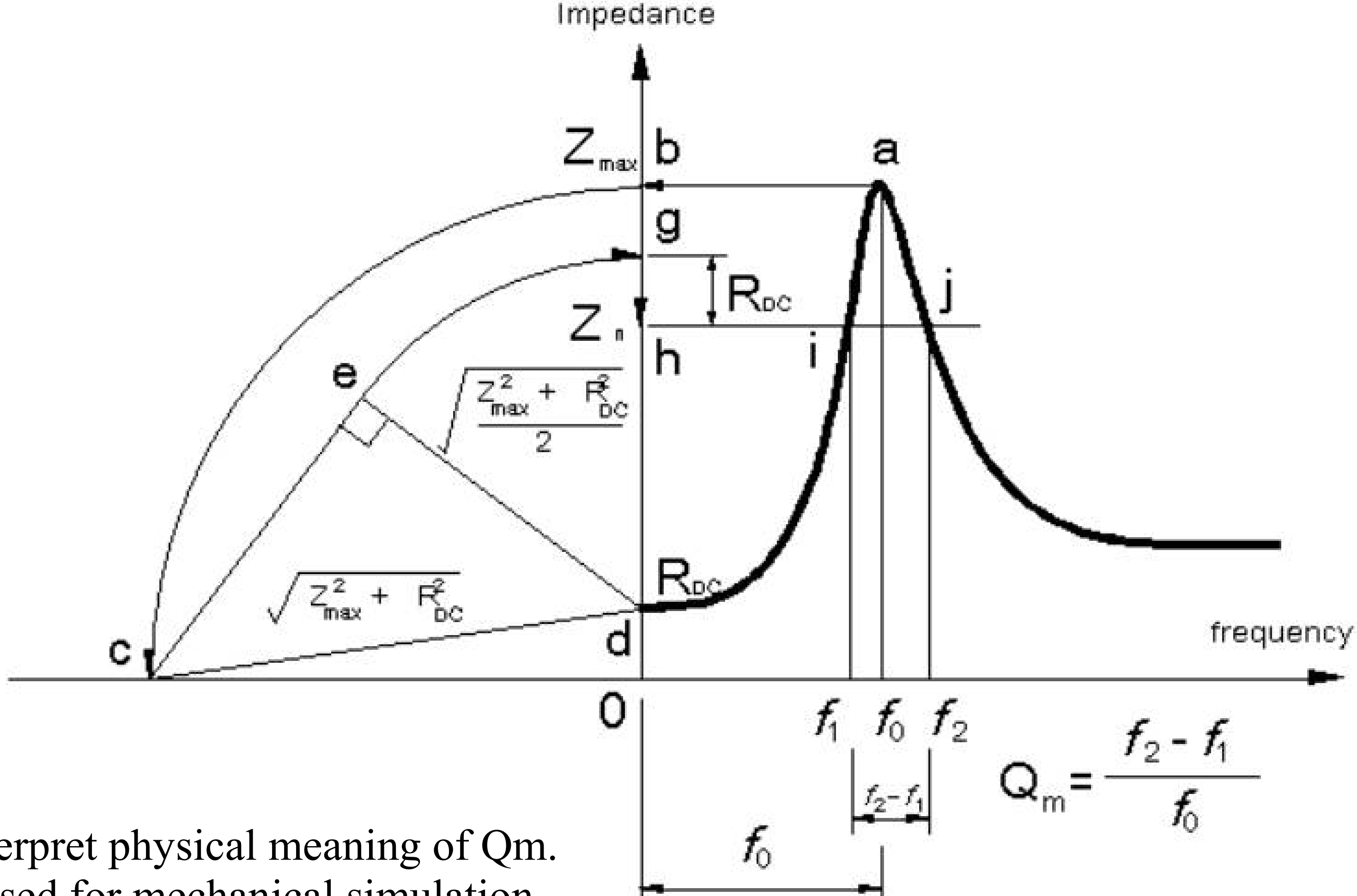
Required parameters for mechanical simulation

	Physical properties for this simulation	Parameter of Speaker Unit
Speaker Unit	(1) Mass of membrane (2) Spring constant of speaker Unit (ku)	m0 f0 $f_0 = \frac{1}{2\pi}\sqrt{\frac{k_u}{m_0}} \rightarrow k_u = 4\pi^2 f_0^2 m_0$
Speaker Cabinet	(1) Spring constant of chamber from corresponding membrane of speaker unit Volume of Chamber (V0) Effective membrane area (2)Spring constant of chamber from corresponding duct Sectional area of duct	a0 ***Only THREE TS parameters are Required for mechanical simulation***

(3) Mass of air involved in duct
mass = density of air x sectional area of duct x effective length of duct

What is parameter Q?

Geometrical interpretation of Qm



I cannot interpret physical meaning of Qm.
Qm is not used for mechanical simulation.
Is Qm constant a function of input power?

Simulation Example of Single Bass-Reflex (1)

Table1　Specification of FE166Sigma

Parameter	Value	Note
f_0	50[Hz]	
m_0	0.0069[kg]	6.9[g]
a_0	0.013273[m^2]	Effective radius of membrane = 6.5[cm]

Table 2 Values of Parameters

Parameter	Symbol	Value	Unit
Effective membrane area	a_0	0.01327	[m^2]
Cross sectional area of duct	a_1	0.006600	[m^2]
Ratio of duct area vs. membrane area (a_1/ a_0)	r_1	0.00497362	[-]
Effective moving mass of membrane	m_0	0.006900	[kg]
Effective moving mass of air in duct	m_1	0.001323	[kg]
Spring constant of speaker unit	k_u	681.0	[N/m]
Spring constant of chamber corresponding to membrane under adiabatic condition (equithermal condition)	k_0	998.94 (713.53)	[N/m]
Amplitude of driving force	f_A	0.1	[N]
Size of discretized time	δ	0.00001	[s]

Simulation Example of Single Bass-Reflex (2)

Apply characteristic frequency of duct:

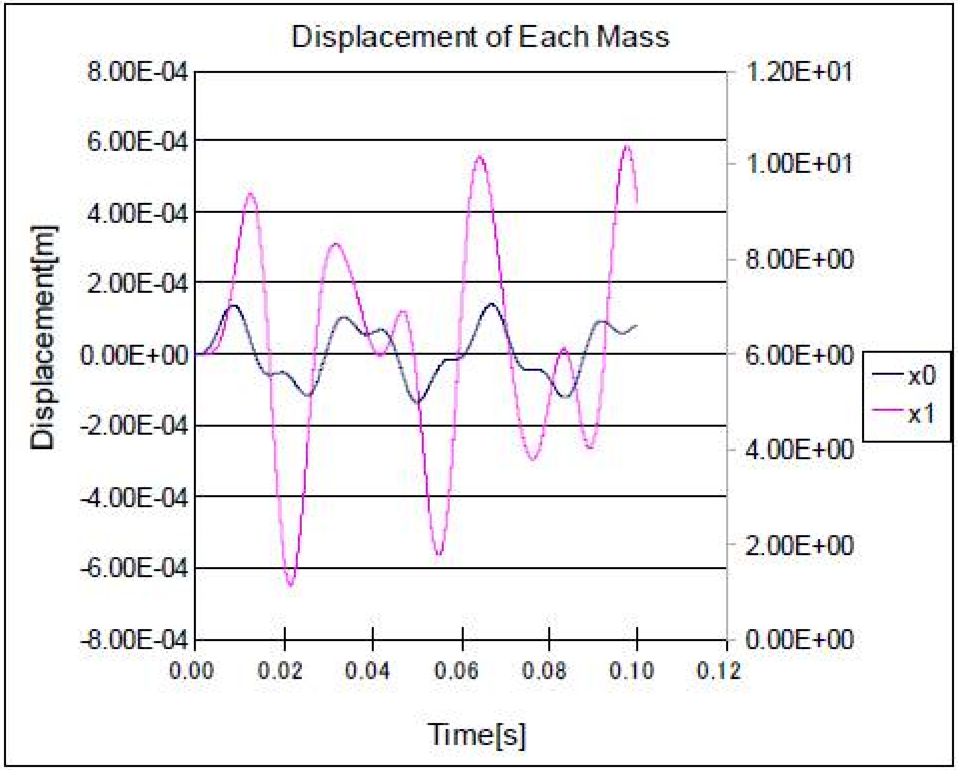


Fig. 3A Displacement of each mass in time series (58.1Hz)

Sine curve was forced, but result was ugly.

Simulation Example of Single Bass-Reflex (2)

Apply lower frequency than characteristic frequency.

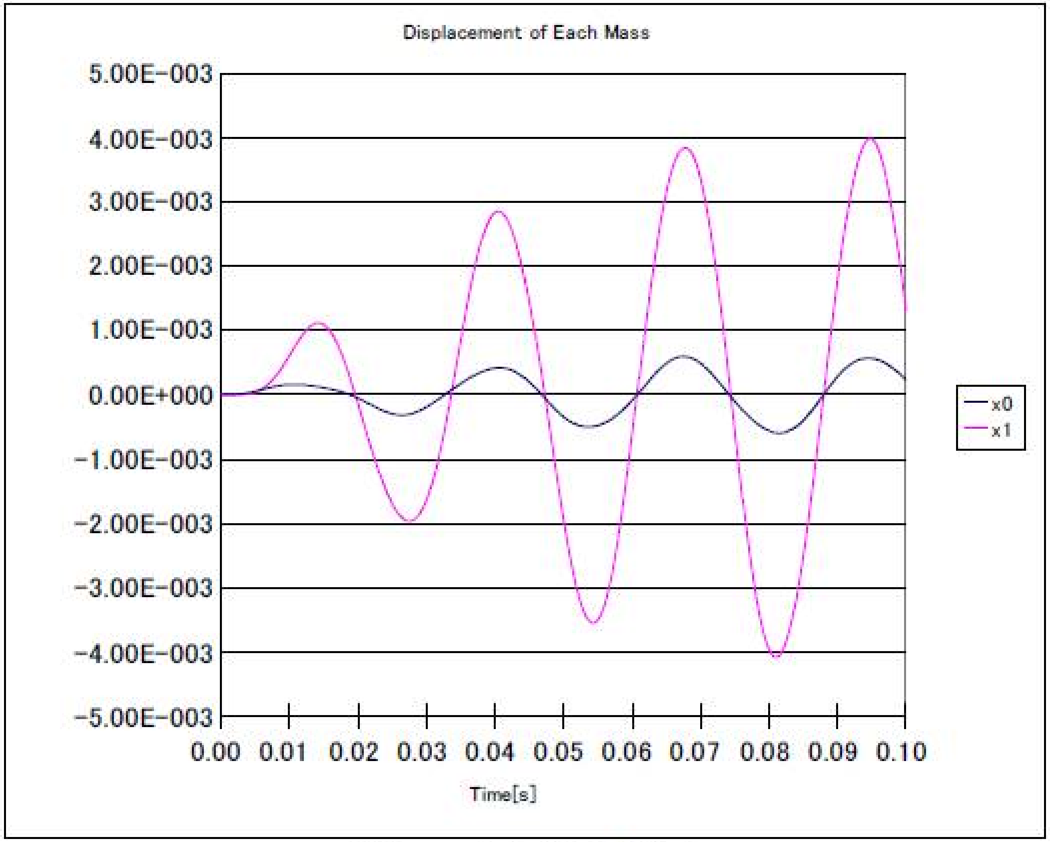


Fig. 4A (40Hz)

Phase of sound from duct and back side of menbrane are same (as we already know)

Simulation Example of Single Bass-Reflex (3)

Apply higher frequency than characteristic frequency.

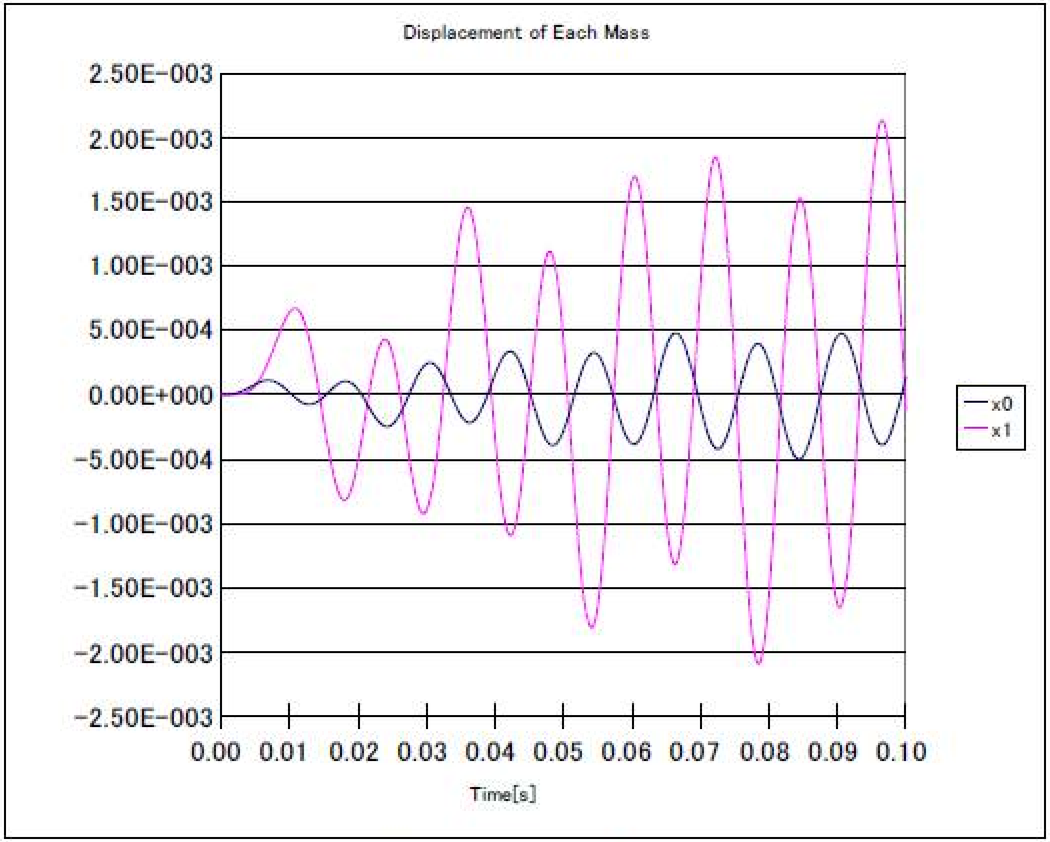


Fig. 5A (80Hz)

Phase became reverse: This frequency is enforced.
Delay of air mass of duct was around 0.01s and it skips one cycle then catches up.

Simulation Example of Single Bass-Reflex (4)

Apply much higher frequency.

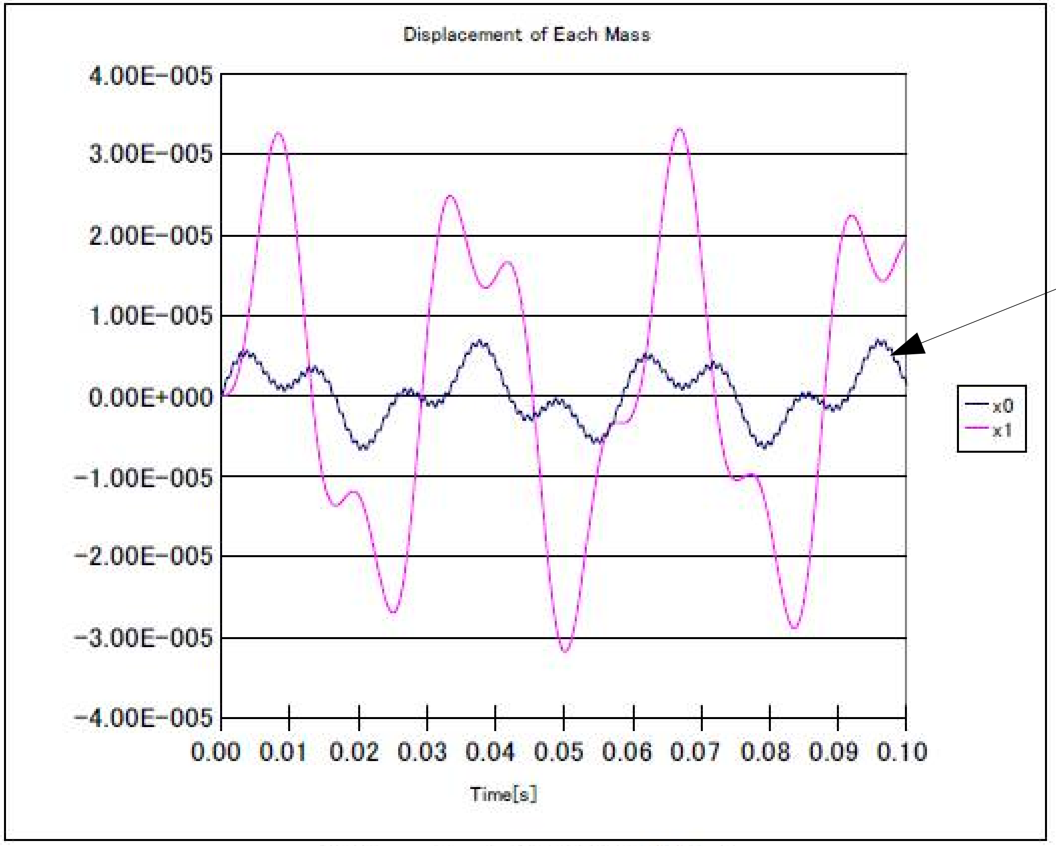


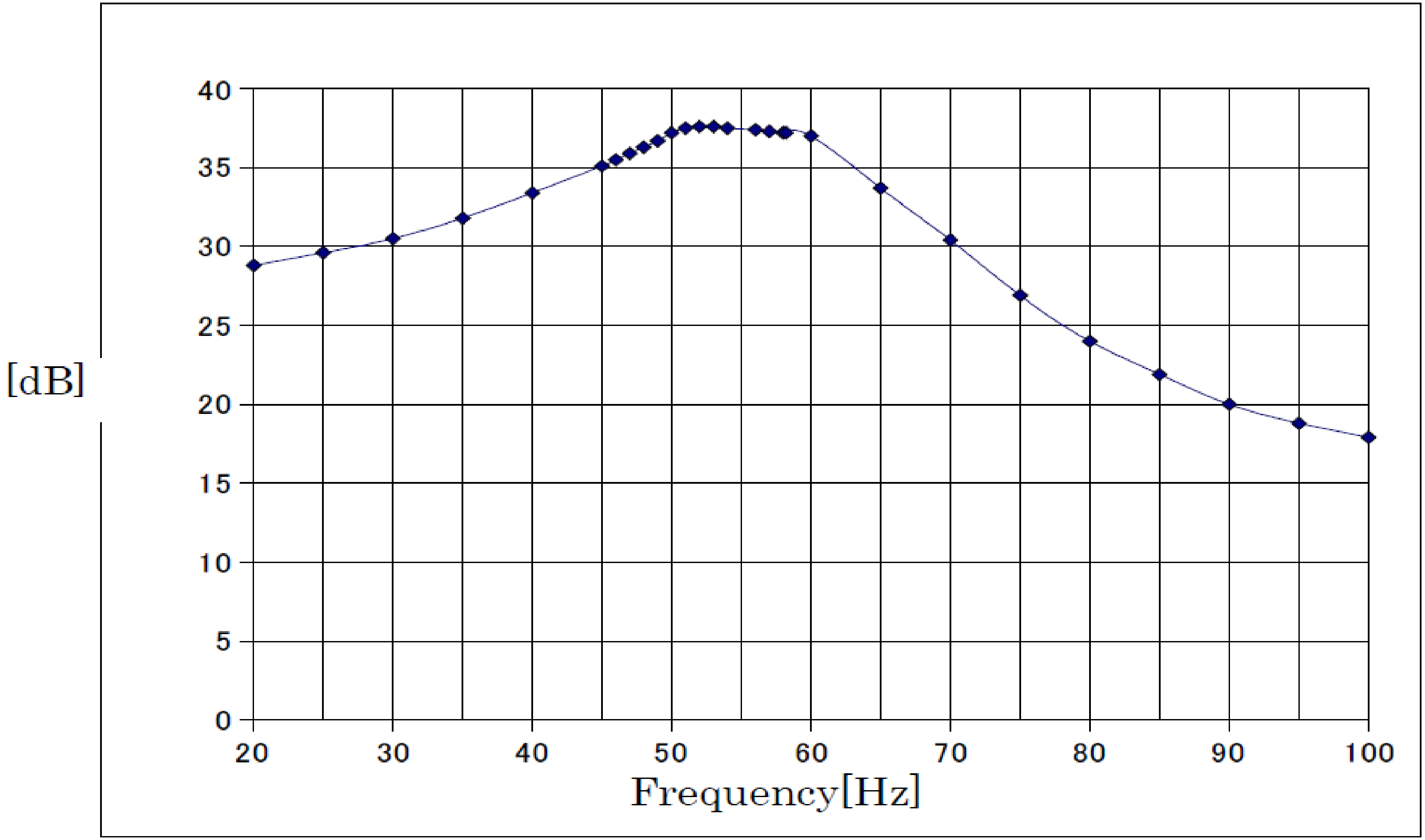
Fig. 6A (1000Hz)

Small fluctuation is forced frequency

Membrane and air mass show not intentional move: this is mechanical noise.

Simulation Example of Single Bass-Reflex (5)

Difference of SPL between duct and membrane



Higher peak shows characteristic resonation.

Questions?

- Why were not multiple-degree of freedom cavity resonators studied?
- Were traditional theories proven enough?
- Were there enough mechanical simulation in the past?
- How do we execute driving system simulation? (I have no answer. Other TS parameters will be used for electrical simulation.)
- Did you know that loudspeaker system generated characteristic noise?